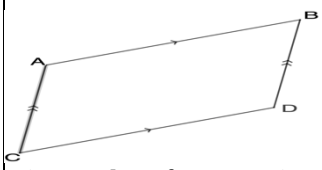
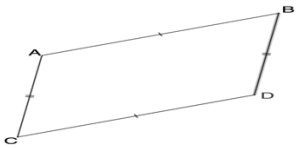
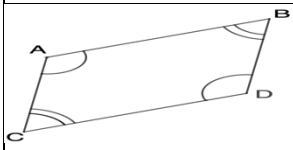
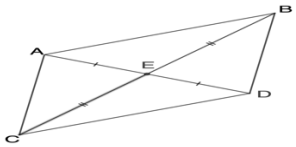
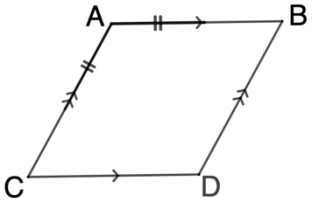
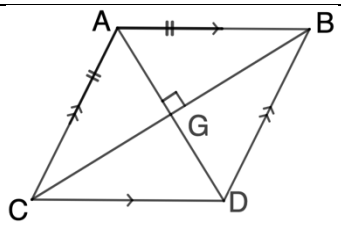
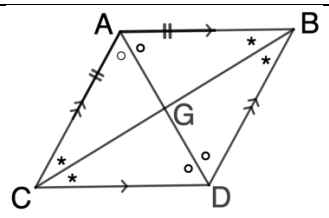
SUBJECT and GRADE	Mathematics Gr 10	
TERM 3	*Week 7*	
TOPIC	Geometry	
AIMS OF LESSON	Solve problems using Midpoint and quadrilateral theorems	
RESOURCES	***Paper based resources***	***Digital resources***
	Go to the Geometry Section in your Textbook	https://www.youtube.com/watch?v=2az9sa0j_R
INTRODUCTION		

In Term 1 you would have investigated various properties of triangles and quadrilaterals, made conjectures and then proved or disproved these conjectures. The conjectures that you were able to prove are known as Theorems. All the properties of the different quadrilaterals you must know as it will enable you to answer other questions. Below is a summary of the properties of quadrilaterals

Study the Table below with all the properties of the various quadrilaterals

Quadrilateral with Properties	Sketches reflecting the properties	
Parallelogram (Is abbreviated as, parm or ∥ m) A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Both pairs of opposite sides are equal in length. Both pairs of opposite angles are equal. Both diagonals bisect each other.	 (opp sides of ∥m are ∥)	 (opp sides of ∥m are =)
	 • (opp ∠s of ∥ m)	 (diag of ∥m)
Rhombus Is a parallelogram that has all four sides equal in length. Both pairs of opposite sides are parallel. All sides are equal in length. Both pairs of opposite angles are equal. The diagonals bisect each other at 90°. The diagonals of a rhombus bisect both pairs of opposite angles.		 (diag of rhombus)  (diag of rhombus)

Rectangle

Is a parallelogram that has all four angles equal to 90°

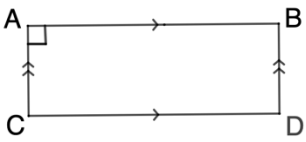
Both pairs of opposite sides are parallel.

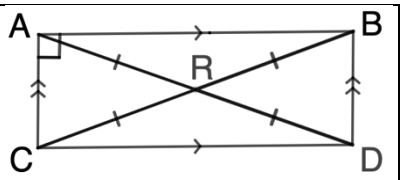
Both pairs of opposite sides are equal in length.

The diagonals bisect each other.

The diagonals are equal in length.

All interior angles are equal to 90°.





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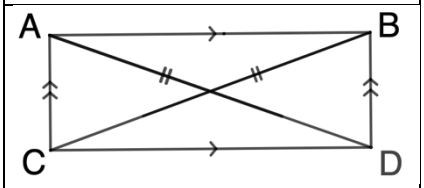


(diags of rect)

Square:

Is a quadrilateral with all sides and angles equal.

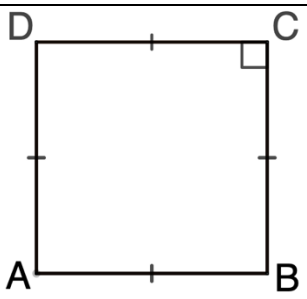
Is a rhombus that has all four interior angles equal to 90°.

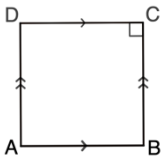
Both pairs of opposite sides are parallel.

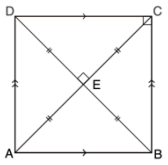
The diagonals bisect each other at 90°.

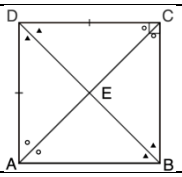
The diagonals are equal in length.

The diagonals bisect both pairs of interior opposite angles (i.e. all are 45°)









Kite:

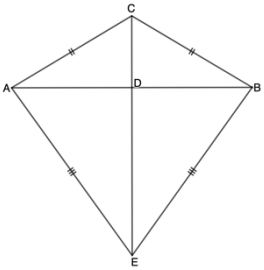
Is a quadrilateral with two pairs of adjacent sides equal.

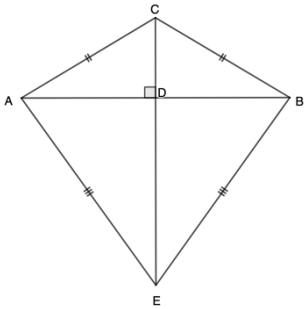
One pair of opposite angles are equal (the angles are between unequal sides).

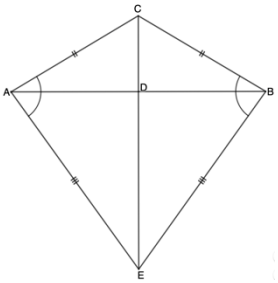
The diagonal between equal sides bisects the other diagonal.

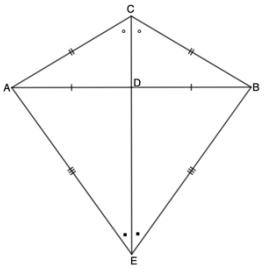
The diagonal between equal sides bisects the interior angles.

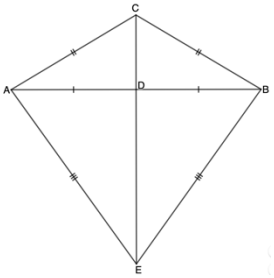
The diagonals intersect at 90°.



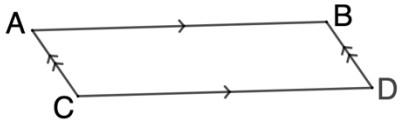




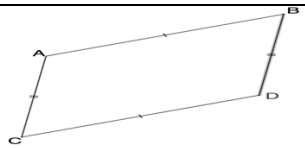




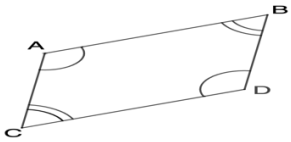
To Prove a Quadrilateral is a parallelogram you have to prove one of the following:



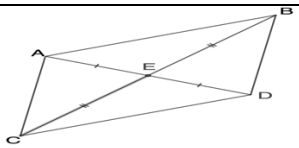
Both pairs of opposite sides of the quadrilateral are equal.
(opp sides of quad are ‖)



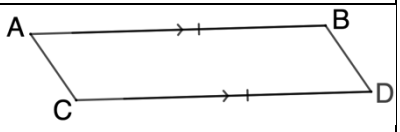
Both pairs of opposite sides of the quadrilateral are equal.
(opp sides of quad are =)



Both pairs of opposite angles of the quadrilateral are equal.
(opp angles of quad are =)

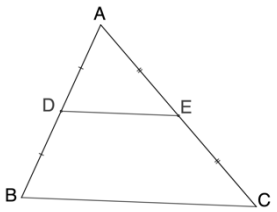
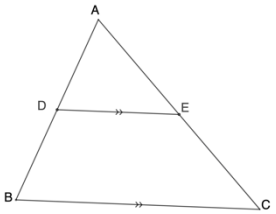
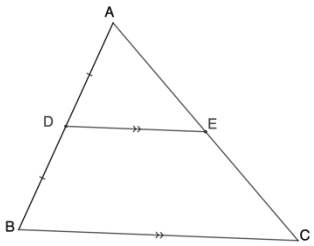
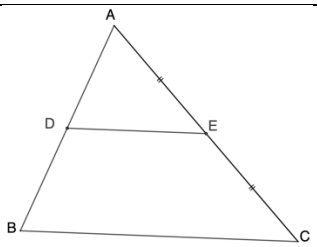


The diagonal of quadrilateral bisect each other.
(diag of quad bisect each other)



A pair of opposite sides are equal and parallel.
(pair of opp sides = and ‖)

Note a theorem is a statement, giving some information which if that is is the case you will be able to deduce some other information. A theorem can be broken up as in the table below.

Theorem	*If*	Then
A line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side. (Midpt theorem)		
The Line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side. (line through midpt‖ to 2nd side)	 • •	

In answering Geometry questions you will be required to combine two or more statements to draw conclusions. Look at the following examples.

Example 1

Given: A = B + C , B = Q + R and C = Q what can you conclude?

Solution:

A = B + C

∴ A = Q+R + C ∵ B = Q + R

∴ A = C + R + C. ∵ C = Q

∴ A= 2C + R

CAN YOU?

1. Given: A = B and B = C what can you conclude?
2. Given: A = B + C and B = C what can you conclude?
3. Given: A + B = 180° , C= A and D = B what can you conclude?
4. Given: A = B+C and A = P+C what can you conclude?

Answer:

1. A = C 2. A = 2C. 3. C + D = 180° 4. B = P

In Geometry you will be asked to prove or conclude a particular statement and you will have to determine what are the required statements that will logically lead to that conclusion.

Example 2: Given: C + D = 90° what additional information will you require to conclu A + D = 90°?

This is a very important skill to practice

Solution:

C + D = 90°

So to conclude that

A + D = 90

You will need to show that A= C

CAN YOU?

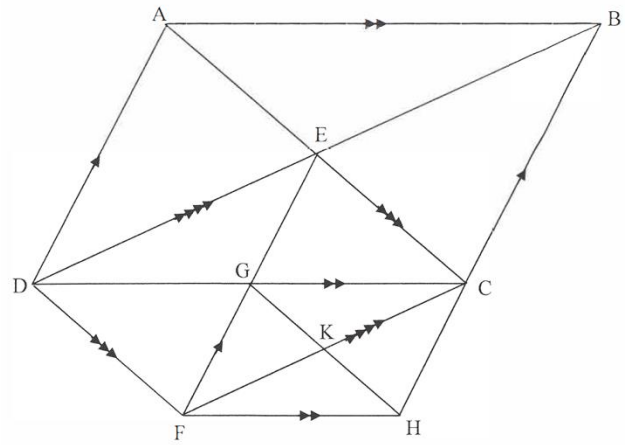
5. Given: P = Q + R what additional information will you require to conclude that Q = S°?
6. Given: A + C=180° what additional information will you require to conclude that A = B?
7. Given: C=F what additional information will you require to conclude that C = D?

Answer:

5. P = Q + R 6. A + B =180°. 7. D = F

Example 3: In the diagram, ABCD is a parallelogram with diagonals intersecting at E. The diagonals of parallelogram DECF intersect at G. The diagonals of parallelogram FGCH intersect at K.

Prove that DB = 4KC



The approach in doing a Geometry questions is vital:

1. Read the question carefully, before filling in too much information. Too much information can lead to confusion. You could use a different colour to indicate each of the different parallelograms
2. Note what you need to prove, DB is the diagonal of the big parallelogram ABCD. KC is the diagonal of the small parallelogram FGCH.
3. Think about the theorems involving the diagonals of a parallelogram.
4. Diagonals of parallelogram bisect each other. That means, DE=EB and DB= DE + EB.
   ∴ DB = 2 DE
5. Observe that DE is the side of the parallelogram DECF. What theorem to you know involving the sides of a parallelogram? Opposite sides of a parallelogram are equal. Therefore DE= FC. And FC = FK+ KC, we also know FK= KC. Therefore FC= 2KC, we took KC instead of FK because KC is in the question that must be answered. So DB = 2DE, DE = FC and FC=2KC. Combining these statements we get DB=4KC. Now we will write out the solution.

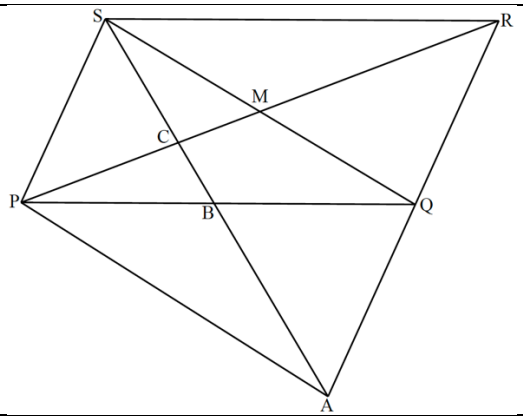
Solution:

Statement	**Reason**
DE = EB	diag of ∥m
DE = FC	opp ∠s of ∥ m
KC=FK	diag of ∥m
∴FC=2KC	
∴DB=2DE **∴DB=2FC=2(2KC)=4 KC**	

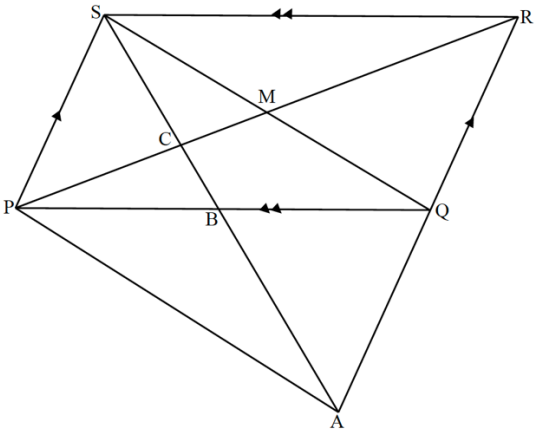
CAN YOU?

In the diagram alongside, PQRS is a parallelogram having diagonals PR and QS intersecting in M. APSQ is a parallelogram with diagonals PQ and SA intersecting at B. C is the point of intersection of SA and PR. SP = 2 SM

Prove that AR = 4 SM



Example 4: In the diagram below, PQRS is a parallelogram having diagonals PR and QS intersecting in M. B is a point on PQ such that SBA and RQA are straight lines and SB = BA. SA cuts PR in C and PA is drawn.



a) Prove that SP = QA.
b) Prove that SPAQ is a parallelogram.
c) Prove that AR = 4MB.

Solutions:

Statement	**Reason**
a) In ΔPSB and ΔQAB	
1. SB = AB	Given
2. $S\hat{B}P = A\hat{B}Q$	vert opp $\angle s$
3. $P\hat{S}B = Q\hat{A}B$	alt ∠, PS∥AR
∴ $\Delta PSB \equiv \Delta QAB$	∠ ∠ S
∴ SP = QA	
b) SP=QA	From (a)
SP∥ QA	Opp sides of ∥m
SPAQ is a ∥m	pair of opp sides = and ∥
c) AR = 2 AQ	AQ= PS and PS=QR
MB= $\frac{1}{2}$AQ	M is midpt SQ
∴ AQ = 2 MB	M is midpt of SA
∴ AR = 2AQ	**Midpt theorem**
∴ AR = 2(2MB)	
∴ AR= 4MB	

a) Colour the sides of parallelogram PQRS. Mark the two sides you trying so show equal. Note that the side SP is a side of parallelogram PQRS. So if we could prove that SPAQ is a parallelogram then SP= QA. Notice that in question(b) you have to prove SPAQ is a parallelogram. So we need to find another route. Notice that SB= BA is given. Indicate these two equal sides in the same colour. You will now notice that SP and SB are the sides of ΔPSB. Similarly QA and AB are the sides of ΔQAB.
If we can prove $\Delta PSB \equiv \Delta QAB$. Then SP = QA. So we already have one side in each triangle which is equal.

b) The cases of congruency are, SSS, S included∠ S, ∠ ∠ S, 90° hyp S.

c) Two angles can be determine equal to two angles in the other triangle because of vertically opposite angles and alternate angles.

d) We may now proceed to write out the solution, ensuring that all the steps follow logically with reason.

Please note there is often more than one solution to a question. It has to do with which theorems come to mind and you start using. Below is another solution to question (a) of example 4.

Statement	Reason
B is midpoint of AS BQ‖ SR ∴ SP = QA	Given, SB = BA Opp sides of ‖m line through midpt‖ to 2nd side

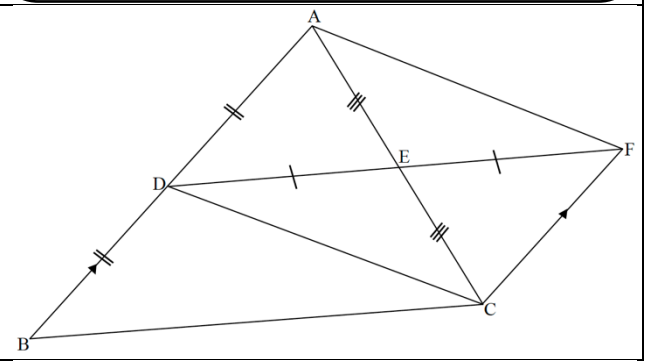
If you identified that B was the midpoint of AS and that BQ‖SR, you would have been abel to apply the theorem that "a line segment from midpoint of a side of a triangle parallel to the second side bisects the third and then conclude that SP=QA.

CAN YOU?

8.

In the diagram alongside, D is the midpoint of side AB of ΔABC. E is the midpoint of AC. DE is produced to F such that DE = EF. CF | | BA.

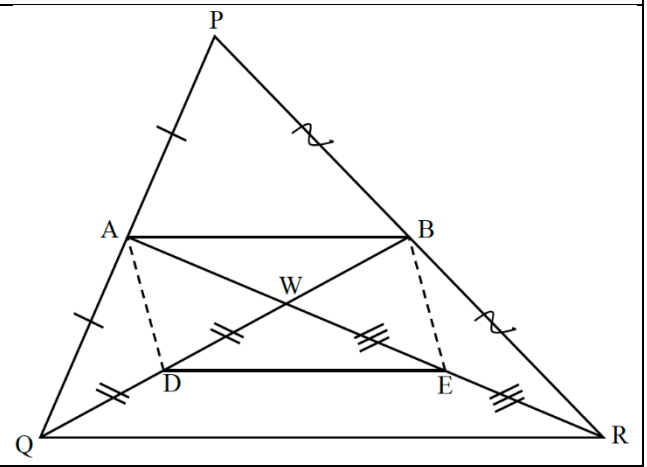
a) Write down a reason why ΔADE ≡ ΔCFE.
b) Write down a reason why DBCF is a parallelogram.
c) Give the reason why $DE = \frac{1}{2}BC$



9.

In ΔPQR, A and B are the midpoints of sides PQ and PR respectively. AR and BQ intersect at W. D and E are points on WQ and WR respectively such that WD = DQ and WE = ER.

Prove that ADEB is a parallelogram



ACTIVITIES/ ASSESSMENT

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev	205	1	248	10.5*	178*	14.1	331	12.1	405
Some	206	2	249			14.2	335	12.2	409
Ch		Rev	251			14.3	337		
						14.4	338		
						14.5	341		

CONSOLIDATION

Ensure that you know all the properties of a parallelogram, rectangle, rhombus, square and kite.

Also ensure you know how to prove that a particular quadrilateral is a Parallelogram, rectangle, rhombus or kite.